Modelling the Effect of Concentration on Non-Newtonian Apparent Viscosity of an Aqueous Polyacrylamide Solution

S. E. E. Hamza

Physics Department, Faculty of Science, Benha University, Benha, Egypt

Abstract

The rheological properties of Polyacrylamide (PAA) solution are extremely important to know if one is designing a system that will extrude the polymer into various shapes. The present study is to determine the rheological behaviour of PAA aqueous solutions at different concentrations (0.25%–1%) by using Brookfield DV-II rheometer. Various rheological models are reviewed for a better understanding of their applicability to the PAA solution in determining the viscosity. The models that have been investigated are Jeffrey's model, Oldroyd 8-constant model and Giesekus model. The aforementioned models are the most powerful for predicting the rheological behaviour of viscoelastic fluids. For each model, the parameters which yield the best fit with experimental data are determined. Results showed that, Giesekus model provide the best prediction of viscosity as a function of shear rate and the effects of concentration can be adequately described by an exponential function. For adequate prediction of non-Newtonian viscosity as a function of both shear rate and concentration, simple correlations were proposed. Comparing the results with experimental data revealed that the proposed correlations are in good agreement with apparent viscosities.

Indexing terms/Keywords

Rheological behaviour, Viscoelastic models, Polyacrylamide, Apparent viscosity.

Academic Discipline and Sub-Disciplines

Physics, Fluid Mechanics

SUBJECT CLASSIFICATION

Rheology of Polymers

TYPE (METHOD/APPROACH)

Theoretical Study.

INTRODUCTION

Rheology is the science that studies the properties and the mechanical behaviour of bodies subjected to a deformation or flow caused by shear stress. Rheological properties are one of the most important factors which are required in the design of different industrial processes. It is also important in the formation of polymeric material in preparing them for fabrication processes [1, 2]. Rheology is involved in many other aspects of polymer science. A flow model may be considered to be a mathematical equation that can describe rheological data, such as viscosity versus shear rate and that provides a convenient manner of describing the data. In addition to mathematical convenience, it is important to quantify how magnitudes of model parameters are affected by structure (concentration) and establish widely applicable relationships that may be called functional models. Understanding the solution rheology of polymers is a key factor in developing suitable models for the solution polarization process [3, 4].

Although there have been numerous efforts to develop viscoelastic models, any of the existing models is still far from complete in terms of being able to explain the wealth of complicated phenomena occurring in flowing chain-molecule systems [5-7]. However, many models do have merit and can provide some clues that are useful in interpreting complex phenomena and possibly even predicting behaviour which is unknown to us. Therefore, it is very important to understand the advantages and disadvantages of the existing rheological models through detailed physical and mathematical analyses and comparison with an experimental data.

Rheological models may be grouped under the categories: (1) empirical, (2) theoretical, and (3) structural [8]. Obviously, an empirical model, such as the power law, is deduced from examination of experimental data. A theoretical model is derived from fundamental concepts and it provides valuable guidelines on understanding the role of structure. It indicates the factors that influence a rheological parameter. A structural model is derived from considerations of the structure and often kinetics of changes in it. Theoretical models of polymers are based on a differential or integral representation. From a mathematical point of view, the differential representation is easier to handle than the integral one. The characteristic feature of linear viscoelastic materials is that the stress is linearly proportional to the strain. Linear viscoelasticity is applicable for small deformations, low rate and low stress. Nonlinear viscoelastic behaviour is exhibited when the deformation is large [8, 9]. Consequently, nonlinear viscoelastic mathematical models are needed [10]. Existing nonlinear mathematical rheological models are constructed through modifications and extensions to higher-order stress or strain terms of the linear models [11].

The measurement of the rheological properties of PAA solutions becomes complicated at very low PAA concentrations (i.e., a few parts per million). Under these conditions, the rheological behaviour of these solutions is not understood, because of the measurement of these systems falls below the range of sensitivity of rheometers [12]. Therefore, PAA solutions are studied at polymer concentrations above 0.1% [13].

In this work, the effects of concentration on the rheological properties of PAA solutions are investigated. We carry out an analysis on three simple viscoelastic fluid models, each of which has a reasonably sound physical basis: Jeffreys, Oldroyd 8-constant and Giesekus's models. Polyacrylamide solutions were chosen as the test material because it exhibits a strong viscoelastic characteristic even at low concentration. Furthermore, it has a very broad scope of industrial applications. This type of polymer has been used effectively as film former in industrial waste treatment, in petroleum industry and in many other process industries.

Since the rheological response of PAA solutions to an imposed stress is sensitive to its concentration, Brookfield DV-II rheometer is used to measure the dependence of viscosity on shear rate at different fluid concentrations. In the present paper, we picked up typical viscoelastic models and studied them by comparing computed values with experimental data of the fluid apparent viscosity.

GOVERNING EQUATIONS

For steady incompressible flow, the equations governing the flow are the continuity and the momentum equations:

$$\nabla \cdot \underline{V} = 0 , \tag{1}$$

$$\rho\left(\frac{\partial \underline{V}}{\partial t} + \underline{V} \cdot \nabla \underline{V}\right) = -\nabla p + \nabla \cdot \underline{\underline{\tau}} , \tag{2}$$

where $\underline{V}$, ρ, p and $\underline{\underline{\tau}}$ are the velocity, density, pressure and stress tensor which is given for Newtonian fluid as:

$$\underline{\underline{\tau}} = 2\eta \underline{\underline{d}} , \tag{3}$$

Where η is a (constant) viscosity and $\underline{\underline{d}} = \frac{1}{2}\left(\underline{\underline{L}} + \underline{\underline{L}}^T\right)$ is the rate of deformation tensor with $\underline{\underline{L}} = \nabla \underline{V}$. Substituting of equation 3 in the momentum equation, equation 2, leads to the well known Navier-Stokes system. As already discussed, this set of equations is commonly used to describe Newtonian fluids but polymers exhibits relevant non-Newtonian characteristics and more complex constitutive models should be used.

Convected Derivatives

The viscosity of PAA solutions shows the shear-thinning property. Therefore our examinations are restricted to the constitutive equations which describe the shear-thinning. These models are written in a frame of reference that is convected or deformed with the material elements. There are two convected coordinate systems. In one, the base coordinate vectors are parallel to material lines; they are stretched and rotated with the material lines. With these base vectors, we have the upper-convected time derivative for the tensor $\underline{\underline{\tau}}$. The second choice is a set of base vectors that are normal to material planes; in a deformation each base vectors remain normal and stretches so that their lengths remain proportional to the area of the material plane to which it is normal [6]. With these base vectors, we have the lower-convected time derivative for $\underline{\underline{\tau}}$.

The general expression for a one-parameter family of convected derivatives of $\underline{\underline{\tau}}$ is given by:

$$\underline{\underline{\tau}}_{(1)} = \frac{D\underline{\underline{\tau}}}{Dt} - \underline{\underline{\tau}} \cdot \underline{\underline{\omega}} - (\underline{\underline{\tau}} \cdot \underline{\underline{\omega}})^T + a\left(\underline{\underline{d}} \cdot \underline{\underline{\omega}} + (\underline{\underline{d}} \cdot \underline{\underline{\omega}})^T\right), \tag{4}$$

where $\underline{\underline{\omega}} = \frac{1}{2}[\underline{\underline{L}} - \underline{\underline{L}}^T]$ is the vorticity tensor. Notice that, choices of body-fixed frames will yield different objective derivatives. A few classical examples of possible objective convected derivatives $\underline{\underline{\tau}}_{(1)}$ are shown in Table 1. The particular values $a = 1$, 0 and -1 correspond respectively to the lower-convected, co-rotational and upper-convected Maxwell or Oldroyd-type (Oldroyd-A and Oldroyd-B) models.

Table 1. Commonly used convected derivatives

Name	Notation	Parameter (a)	Definition
Lower-convected	$\overset{\Delta}{\underline{\underline{\tau}}}$	1	$\overset{\Delta}{\underline{\underline{\tau}}} = \frac{D\underline{\underline{\tau}}}{Dt} - \underline{\underline{L}}\cdot\underline{\underline{\tau}} - (\underline{\underline{L}}\cdot\underline{\underline{\tau}})^T$
Upper-convected	$\overset{\nabla}{\underline{\underline{\tau}}}$	-1	$\overset{\nabla}{\underline{\underline{\tau}}} = \frac{D\underline{\underline{\tau}}}{Dt} - \underline{\underline{\tau}}\cdot\underline{\underline{L}} - (\underline{\underline{\tau}}\cdot\underline{\underline{L}})^T$
Co-rotational (Jaumann)	$\overset{o}{\underline{\underline{\tau}}}$	0	$\overset{o}{\underline{\underline{\tau}}} = \frac{D\underline{\underline{\tau}}}{dt} - \underline{\underline{\tau}}\cdot\underline{\underline{\omega}} - (\underline{\underline{\tau}}\cdot\underline{\underline{\omega}})^T$

Co-rotational Jeffrey Model

In this model, the relationship between the extra stress tensor $\underline{\underline{\tau}}$ and the rate of deformation tensor $\underline{\underline{d}}$ is given by:

$$\underline{\underline{\tau}} + \lambda_1 \overset{o}{\underline{\underline{\tau}}} = 2\eta_o(\underline{\underline{d}} + \lambda_2 \overset{o}{\underline{\underline{d}}}), \tag{5}$$

where λ_1 and λ_2 are two constants, having the dimension of time $(\lambda_1 \geq \lambda_2 \geq 0)$, referred to as relaxation and retardation time, respectively. Also η_o is the zero-shear-rate viscosity. The model (5) reduces to a pure Newtonian fluid when $\lambda_1 = \lambda_2 = 0$. The Jeffrey's equation, generalized by Jaumann, yields the following dependence of the apparent viscos ity $\eta(\dot{\gamma})$ on the shear rate $\dot{\gamma}$; [6, 8]:

$$\eta(\dot{\gamma}) = \eta_o \frac{1 + \lambda_1\lambda_2\dot{\gamma}^2}{1 + \lambda_1^2\dot{\gamma}^2}, \tag{6}$$

so, the co-rotational Jeffrey's model is characterized by three parameters η_o, λ_1 and λ_2. It follows from expression (6) that the limiting viscosity at zero shear rate equals η_o, while limiting viscosity at infinite shear rate is λ_1/λ_2 times smaller:

$$\lim_{\dot{\gamma}\to 0}\eta(\dot{\gamma}) = \eta_o, \qquad \lim_{\dot{\gamma}\to\infty}\eta(\dot{\gamma}) = \frac{\lambda_2}{\lambda_1}\eta_o = \eta_\infty. \tag{7}$$

Oldroyd 8-Constant Model

This model can describe more rheological behaviours than the convected Jeffrey model. The differential form of this model can be written as:

$$\underline{\underline{\tau}} + 2\lambda_1\overset{\nabla}{\underline{\underline{\tau}}} + \lambda_3\left(\underline{\underline{\tau}}\cdot\underline{\underline{d}} + \underline{\underline{d}}\cdot\underline{\underline{\tau}}\right) + \lambda_5(\mathrm{tr}\,\underline{\underline{\tau}})\underline{\underline{d}} + \lambda_6(\underline{\underline{\tau}}:\underline{\underline{d}})\underline{\underline{\delta}} = 2\eta_o\left[\underline{\underline{d}} + \lambda_2\overset{\nabla}{\underline{\underline{d}}} + 2\lambda_4\left(\underline{\underline{d}}\cdot\underline{\underline{d}}\right) + \lambda_7\left(\underline{\underline{d}}:\underline{\underline{d}}\right)\underline{\underline{\delta}}\right], \tag{8}$$

where $\underline{\underline{\delta}}$ is the unit tensor and λ_3 through λ_7 are further material time constants. In this model, the corresponding apparent viscosity $\eta(\dot{\gamma})$ is given as; [6, 8]:

$$\eta(\dot{\gamma}) = \eta_o \frac{1 + \sigma_2\dot{\gamma}^2}{1 + \sigma_1\dot{\gamma}^2}, \tag{9}$$

where

$$\sigma_i = \lambda_i(\lambda_3 + \lambda_5) + \lambda_{i+2}(\lambda_1 - \lambda_3 - \lambda_5) + \lambda_{i+5}\left(\lambda_1 - \lambda_3 - \tfrac{3}{2}\lambda_5\right), \tag{10}$$

with $i = 1, 2$. This model is characterized by eight parameters, η_o and λ_1 through λ_7. In order that the apparent viscosity $\eta(\dot{\gamma})$, equation 9, agree at least qualitatively with experimental data, the condition $0 < \sigma_2 < \sigma_1$ must be satisfied. With eight constants and all the extra terms, considerably more variety in rheological response is possible than for the convected Jeffrey's equation. Several simplified versions of this model, in which special values are assigned to some of the constants, have been used extensively in the literature; these are summarized in Table 2. The usefulness of the Oldroyd equation, with all eight constants, is somewhat limited because dealing with all the terms is tedious and difficult.

Table 2. Models included in Oldroyd 8-constant model

Name of model	Number of constants	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
Oldroyd 6-constant	6						0	0
Oldroyd 4-constant	4			0	0		0	0
Co-rotational Jeffery	3			λ_1	λ_2	0	0	0
Gordon-Schowalter	3	$\frac{\eta_s\lambda_1}{\eta_o}$	$\xi\lambda_1$	$\frac{\xi\eta_s\lambda_1}{\eta_o}$	0	0	0	0

Giesekus Model

The Giesekus model is defined for polymer melts or highly concentrated polymer solutions. The differential form of this model can be written as, [14]

$$\underline{\underline{\tau}} = \underline{\underline{\tau}}_s + \underline{\underline{\tau}}_p \ , \tag{11}$$

$$\underline{\underline{\tau}}_s = 2\eta_s \ \underline{\underline{d}} \ , \tag{12}$$

$$\underline{\underline{\tau}}_p + \lambda_1 \overset{\nabla}{\underline{\underline{\tau}}}_p + \alpha\frac{\lambda_1}{\eta_p}(\underline{\underline{\tau}}_p \cdot \underline{\underline{\tau}}_p) = 2\eta_p \ \underline{\underline{d}} . \tag{13}$$

Here the model is written as a superposition of solvent and polymer contributions $\underline{\underline{\tau}}_s$ and $\underline{\underline{\tau}}_p$, to the stress tensor. The parameters η_s and η_p are the solvent and polymer contributions to the Newtonian or zero-shear-rate viscosity η_o and α denotes the Giesekus mobility factor. The zero-shear-rate viscosity η_o and the retardation time λ_2 are given in terms of η_s, and η_p as:

$$\eta_o = \eta_s + \eta_p ; \qquad \lambda_2 = \lambda_1\frac{\eta_s}{\eta_o} . \tag{14}$$

So, this model is characterized by four parameters η_o, λ_1, λ_2 and α. The inclusion of $(\underline{\underline{\tau}}_p \cdot \underline{\underline{\tau}}_p)$ term in equation 13 gives a viscosity function that is much more realistic than any other model [15]. The apparent viscosity due to this model is given by:

$$\eta(\dot{\gamma}) = \eta_o\left\{\frac{\lambda_2}{\lambda_1} + \left(1-\frac{\lambda_2}{\lambda_1}\right)\frac{(1-f)^2}{1+(1-2\alpha)f}\right\}, \tag{15}$$

where

$$f = \frac{1-\chi}{1+(1-2\alpha)\chi}, \tag{16}$$

and

$$\chi^2 = \frac{\sqrt{1+16\alpha(1-\alpha)(\lambda_1\dot{\gamma})^2}-1}{8\alpha(1-\alpha)(\lambda_1\dot{\gamma})^2} . \tag{17}$$

MATERIALS AND EXPERIMENTAL WORK

In this work, four different concentrations of PAA solution of molecular weight 12.2 million from BLUWAT company, China were prepared. The solutions were prepared by gradual addition of known amount of polymer powder to half litter of distilled water at room temperature. The concentrations ranged from 0.25% to 1.00% with 0.25% increment. Rheological measurements were performed on Brookfield viscometer DV-II and all these measurements was at 293K . A fresh sample was used for each measurement. Multiple samples were used for each experiment. Only the average results are reported here.

RESULTS AND DISCUSSION

Rheological Behaviour

Since the viscosity of PAA solutions is a very important factor, it is necessary to collect all the viscometric data to get a more precise formula which describe the viscosity of PAA solution as a function of shear rate and concentration. Table 3 presents the rheological data for all the studied concentrations. From the data, it could be observed that the apparent viscosity of PAA solutions decreased with the increasing shear rates and increases with increasing its concentrations.

Figures 1 and 2 show the rheological behaviour versus shear rate for an aqueous PAA solution at four different concentrations: 0.25%, 0.50%, 0.75% and 1.00% and at room temperature (293 K). Flow curves indicate that, the flow behaviour of this polymer is non-Newtonian fluid because the flow curves show a shear thinning behaviour. For all PAA concentrations, the shear stress increased with the elevation of the shear rate. Figure 2 shows that as the concentration increase the viscosity increase. The viscosity behaviour of PAA at high shear rate simulate a linear relationship (Newtonian fluid) which indicates that the shear rate has less effect on viscosity while at intermediate and lower shear rate the viscosity of PAA simulate the non-Newtonian fluids.

Table 3. PAA apparent viscosity at different concentrations

Shear rate (s^{-1})	Viscosity (mPa s)				Shear Stress (mPa)			
	0.25%	0.50%	0.75%	1.00%	0.25%	0.50%	0.75%	1.00%
5.5	174.82	358.18	511.68	767.52	961.5	1970.0	2814.2	4221.4
8.9	137.43	281.57	402.24	603.36	1223.1	2506.0	3579.9	5369.9
16.5	100.94	206.79	295.42	443.13	1665.4	3412.1	4874.4	7311.6
27.2	78.61	161.06	230.09	345.13	2138.3	4380.9	6258.4	9387.7
39.9	64.91	132.98	189.97	284.96	2589.8	5306.0	7580.0	11370.0
54.6	55.47	113.64	162.34	243.51	3030.7	6209.2	8870.3	13305.4
71.7	48.42	99.20	141.72	212.58	3471.7	7112.8	10161.1	15241.7
89.3	43.39	88.89	126.99	190.48	3874.4	7937.9	11339.8	17009.8
108.6	39.34	80.61	115.15	172.73	4272.7	8753.8	12505.4	18758.0
129.7	36.00	73.76	105.37	158.05	4669.3	9566.4	13666.3	20499.5
151.5	33.31	68.25	97.49	146.24	5046.5	10339.2	14770.2	22155.4
174.7	31.02	63.55	90.79	136.18	5419.1	11102.6	15860.9	23791.3
199.2	29.05	59.52	85.02	127.53	5786.7	11855.6	16936.6	25404.9
224.9	27.34	56.01	80.02	120.03	6148.6	12597.2	17996.0	26994.0
251.7	25.84	52.95	75.64	113.46	6504.7	13326.6	19038.1	28557.1
279.5	24.52	50.24	71.78	107.67	6854.5	14043.3	20061.9	30092.9
307.7	23.37	47.89	68.41	102.61	7192.0	14734.8	21049.7	31574.5
325.2	22.74	46.58	66.54	99.82	7393.7	15148.0	21640.0	32459.9
344.4	22.09	45.26	64.66	96.99	7608.8	15588.7	22269.6	33404.4
363.8	21.50	44.04	62.91	94.37	7820.2	16021.8	22888.3	34332.4
374.2	21.19	43.42	62.03	93.05	7931.1	16249.2	23213.1	34819.6
385.5	20.88	42.78	61.12	91.68	8050.0	16492.7	23561.0	35341.5
395.0	20.63	42.26	60.38	90.57	8148.6	16694.7	23849.5	35774.3

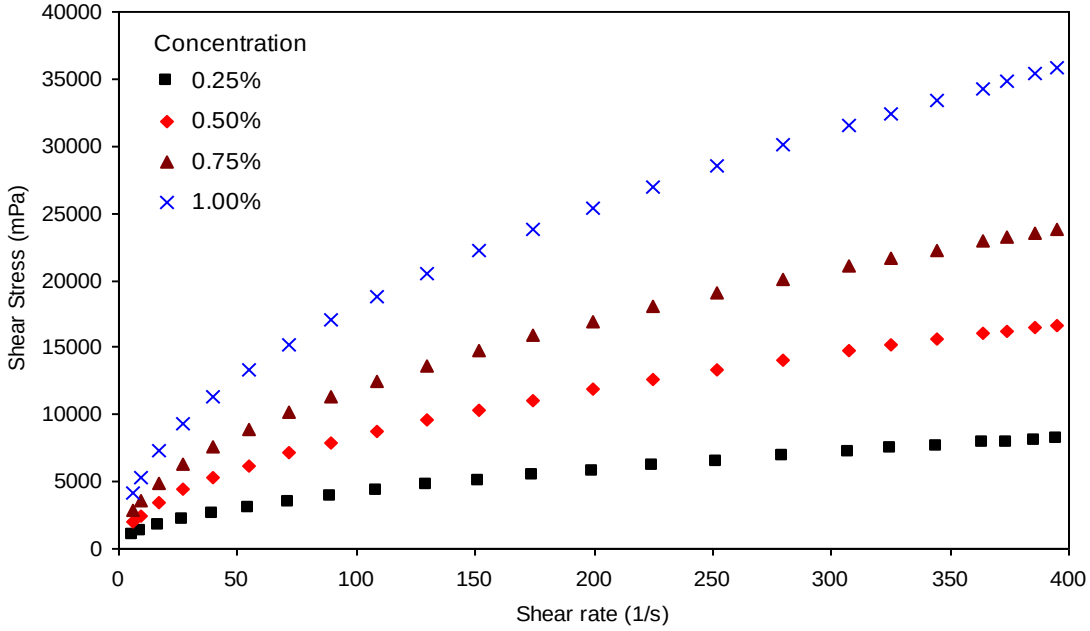

Fig 1: Rheological behaviour of PAA solution at four different concentrations.

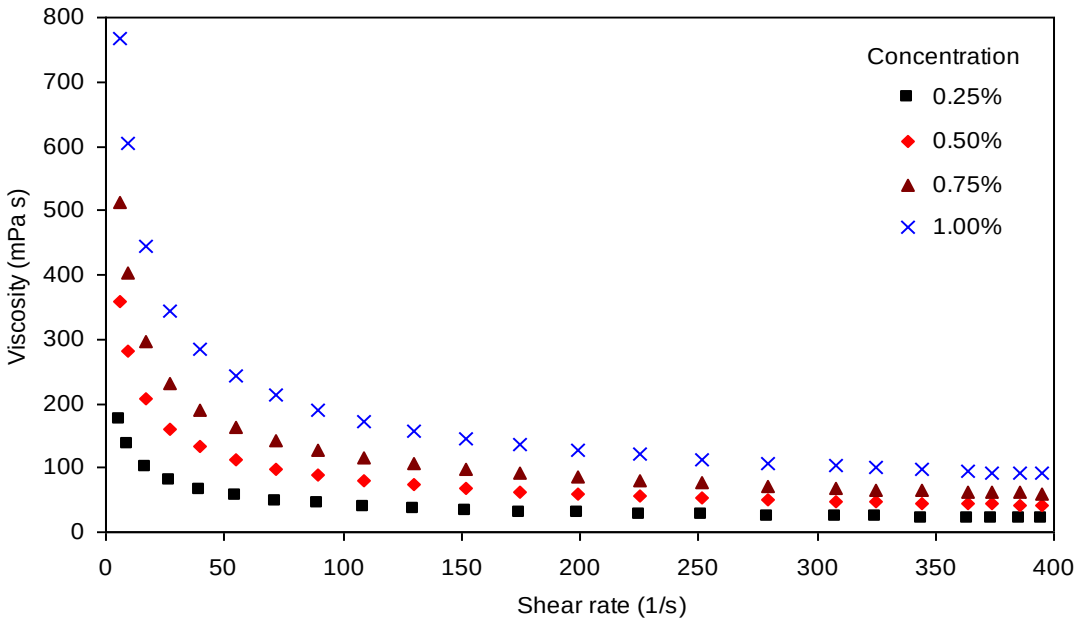

Fig 2: Viscosity curves of PAA solutions at four different concentrations.

Models Fitting

Effect of Shear Rate

The flow behaviour of PAA solutions can be described by the flow curve which is usually presented in the derived function $\eta = f(\dot{\gamma})$. The experimental measurements of the steady shear flow were fitted by using Jeffrey model, Oldroyd 8-constant model and Giesekus model. Tables 4, 5 and 6 show the most suitable parameter values of equations. 6, 9 and 15 respectively.

It is clear from Figures 3 and 4 that, Jeffrey and Oldroyd 8-constant models are not accurately predicts viscosity as a function of shear rate neither at low nor at high shear rate. Figure 5 shows that, Giesekus curve and the experimental points overlap each other or appear to have only a small deviation. This indicates that, the Giesekus model fit is reasonable for the viscosity of PAA solutions.

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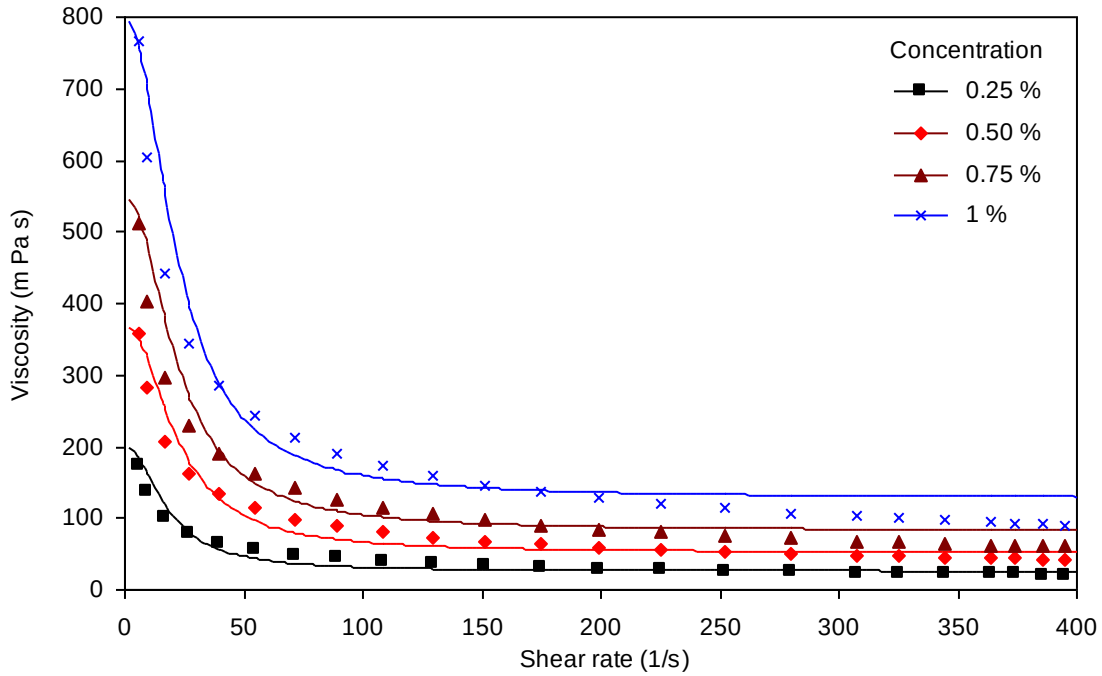

Fig 3: Viscosity-shear rate curves, where dots represent the experimental data and solid lines represent Jeffrey model fit.

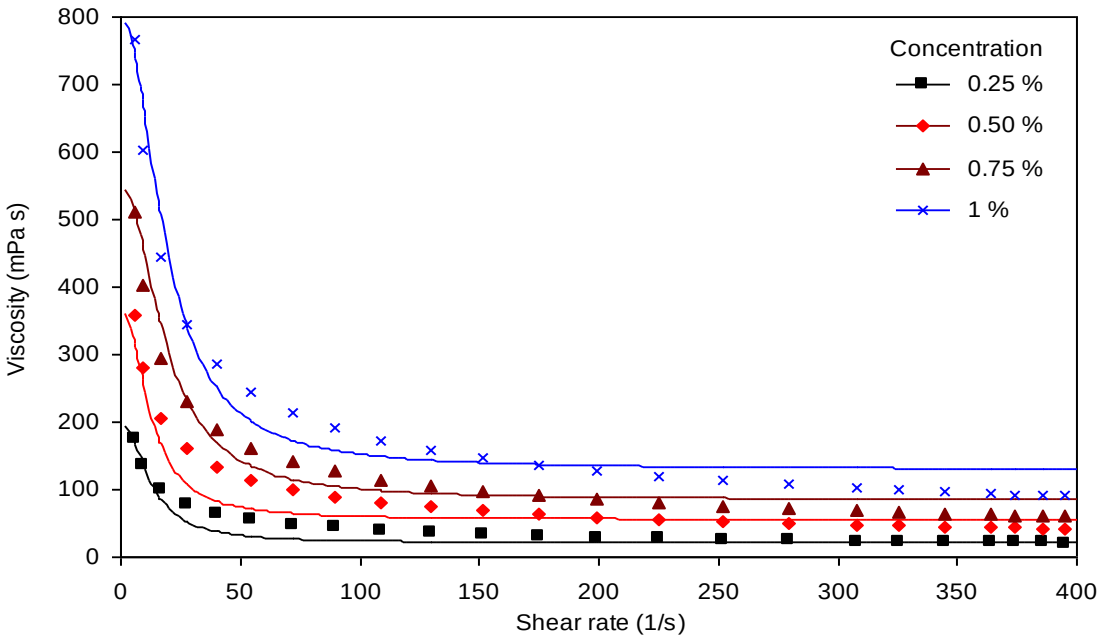

Fig 4: Viscosity-shear rate curves, where dots represent the experimental data and solid lines represent Oldroyd 8-constant model fit.

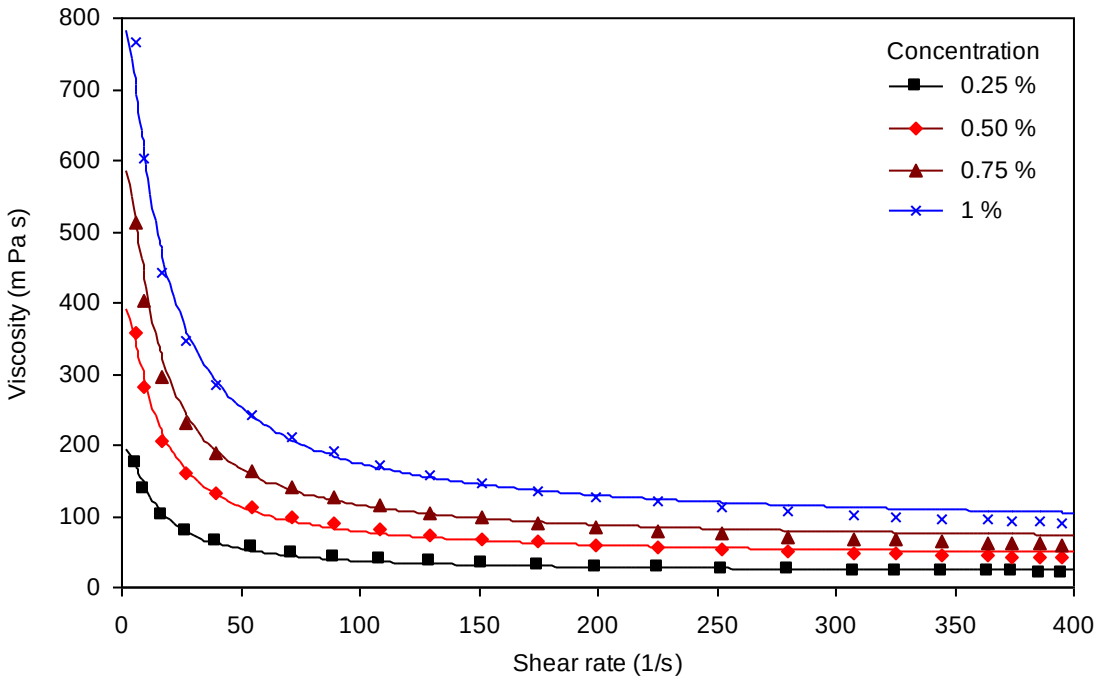

Fig.5: Viscosity-shear rate curves, where dots represent experimental data and solid lines represent Giesekus model fit.

Table 4. Jeffrey's model parameters for PAA solution.

Concentration	η_o (mPa.s)	λ_1 (s)	λ_2/λ_1
0.25%	200	0.055	0.13
0.50%	370	0.045	0.14
0.75%	550	0.045	0.15
1%	800	0.045	0.16

Table 5: Oldroyd 8-constant model parameters for PAA solution.

Concentration	η_o (mPa.s)	λ_1	λ_2/λ_1	λ_3	λ_4	λ_5/λ_1	λ_6	λ_7
0.25%	200	0.09	0.20	0.040	0.008	0.8	0.05	0.020
0.50%	370	0.09	0.20	0.009	0.008	0.8	0.03	0.020
0.75%	550	0.06	0.25	0.009	0.008	0.8	0.03	0.020
1%	800	0.06	0.28	0.009	0.008	0.8	0.03	0.025

Table 6. Giesekus model parameters for PAA solution.

Concentration	η_o (mPa.s)	λ_1 (s)	λ_2/λ_1	α
0.25%	200	0.08	0.1	0.65
0.50%	400	0.08	0.1	0.57
0.75%	600	0.08	0.1	0.5
1%	800	0.08	0.1	0.42

Effect of Concentration

Yong [16, 17] studied the rheological behaviour of PAA solutions. He related the measured viscosity to shear rate and concentration using the following exponential law type:

$$\eta = a_1 \mathrm{Exp}(b_1 C), \tag{18}$$

where a_1 and b_1 are constants and C is the polymer concentration in wt.%. Yong proposed this correlation to predict the viscosity of PAA solution as a function of concentrations. The relationship between the concentration of PAA solution and its flow behaviour at constant shear rates is shown in Figure 6. As seen from the figure, the viscosity increases with increasing PAA concentration and it is sensitive to the lower values of shear rate. It can be observed that at shear rates 374.2, 151.5 and 71.7 s^{-1}, the PAA solutions presented similar behaviours, which in general is an increase for viscosities with the concentration. At shear rate 16.5 and 5.5 s^{-1}, there is an increase for viscosity for concentrations up to 0.5%, going through a sudden increase in viscosity up to 1%. We can state that the concentration functionality of PAA solutions is due to the increase in the molecule interactions between the PAA chains as shown in the literature [18-21].

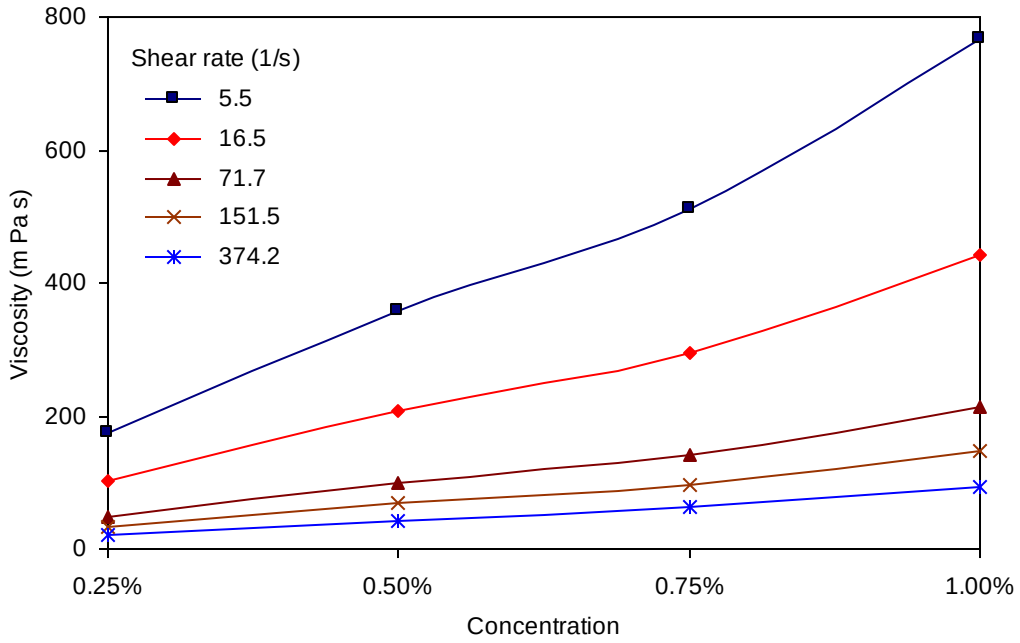

Fig 6: Viscosity behaviour under the effect of concentrations of PAA solutions.

The parameters a_1 and b_1 in equation 18 is calculated from the concentration dependence of $\ln\eta$ at different shear rates, Figure 7. The figure shows that a straight line relation between $\ln\eta$ and the concentration of PAA solution, where the slop is the parameter b_1 and the intersection is $\ln a_1$. Table 7 shows the values of the parameters of the exponential relation, equation 18. The data have a trend line equation:

$$\eta = 280.17\dot{\gamma}^{-0.5}\,\mathrm{Exp}(191.8C), \tag{19}$$

Table 7. Parameters in concentration function for apparent viscosity

Shear rate (1/s)	a_1 (mPas)	b_1
5.5	119.43	191.8
16.5	68.95	191.8
71.7	33.07	191.8
151.5	22.75	191.8
374.2	14.47	191.8

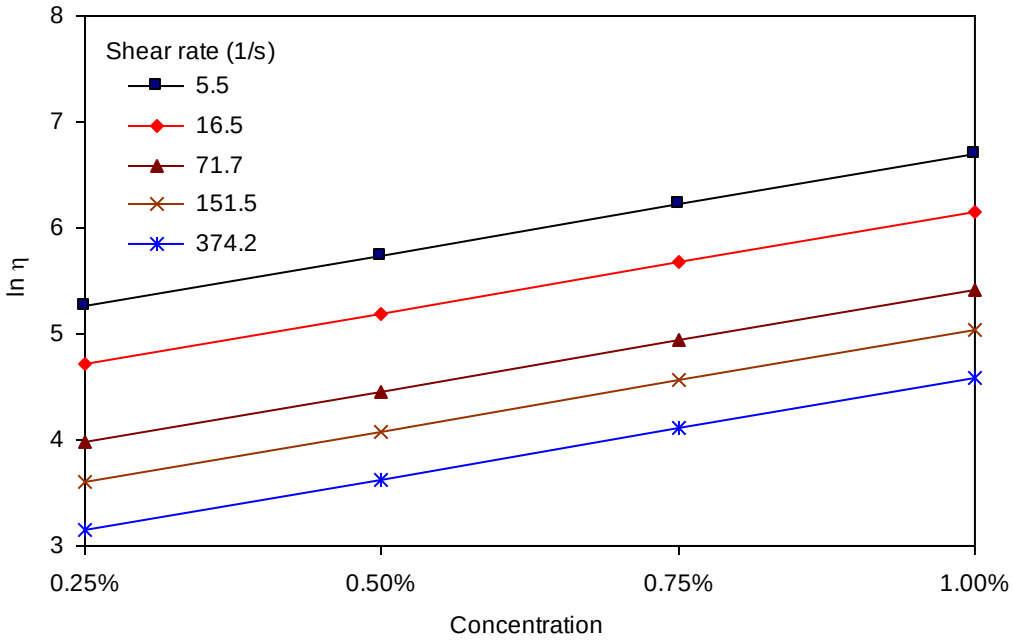

Fig 7 : Logarithmic plots of viscosity versus PAA concentration.

Proposing a New Correlation

As we have seen in the previous sections, Giesekus model accurately predicts the viscosity as a function of shear rate and equation 19 fits the viscosity as a function of concentration. Hence, based on the combined effect of shear rate and concentration, a new correlation is proposed to predict the viscosity of PAA solution. The correlation would be more accurate than previously proposed correlations. The new correlation is as follows:

$$\eta(\dot{\gamma},C) = a_2\,\mathrm{Exp}(b_2\,C)\,\overline{\eta}(\dot{\gamma}), \tag{20}$$

where

$$\overline{\eta} = \frac{\lambda_2}{\lambda_1} + \left(1-\frac{\lambda_2}{\lambda_1}\right)\frac{(1-f)^2}{1+(1-2\alpha)f}, \tag{21}$$

and the parameters λ_1, λ_2 and α as functions of PAA concentration are given in Table 8 and the function "f" in the last equation is given in equation 16. The values of the parameters a_2 and b_2 are calculated. The data have a trend line equation, $172.25\,\mathrm{Exp}(156.413\,C)$. Therefore, the final form of the correlation is as follows:

$$\eta(\dot{\gamma},C) = 172.25\,\mathrm{Exp}(156.413\,C)\,\overline{\eta}(\dot{\gamma}). \tag{22}$$

The combined effects of shear rate and concentration on PAA viscosity can be seen in Figure 8 (in three dimensions form).

Table 8: Giesekus model parameters for PAA solution as functions of concentration.

Concentration	η_o (mPa.s)	λ_1 (s)	λ_2/λ_1	α
0.25% - 1%	$80000\,C$	0.08	0.1	$0.725-30.4\,C$

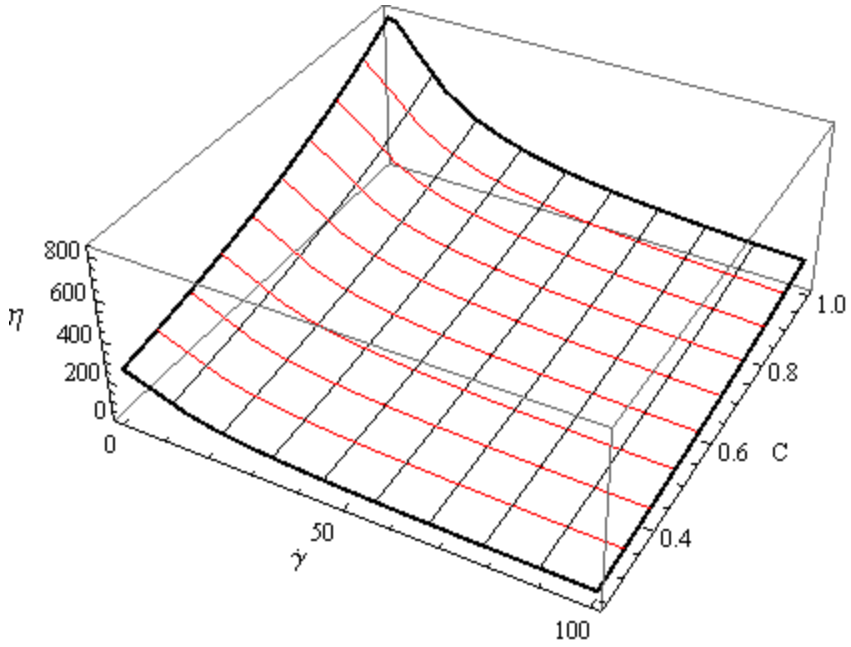

Fig 8 : The combined effects of $\dot{\gamma}$ and C on η of PAA solution.

In order to evaluate the accuracy of this correlation, comparison between the proposed model, equation 22, and the experimental viscosity data at different concentrations were plotted in Figure 9. In most measured data, the curve of correlation and the experimental points overlap each other or appear to have only a small deviation. This indicates that, the suggested correlation has a suitable accuracy and equation 22 is reasonable for the viscosity of PAA solutions. Figure 10 shows a comparison between the results of the experimental data and the proposed correlations fit from equation 22 where $\overline{\eta}$ is taken from the three models, Jeffrey, Oldroyd 8-constant and Giesekus model. It is clear that, the results of the predicted model are in excellent correspondence with those of the experimental data in the most range of concentrations.

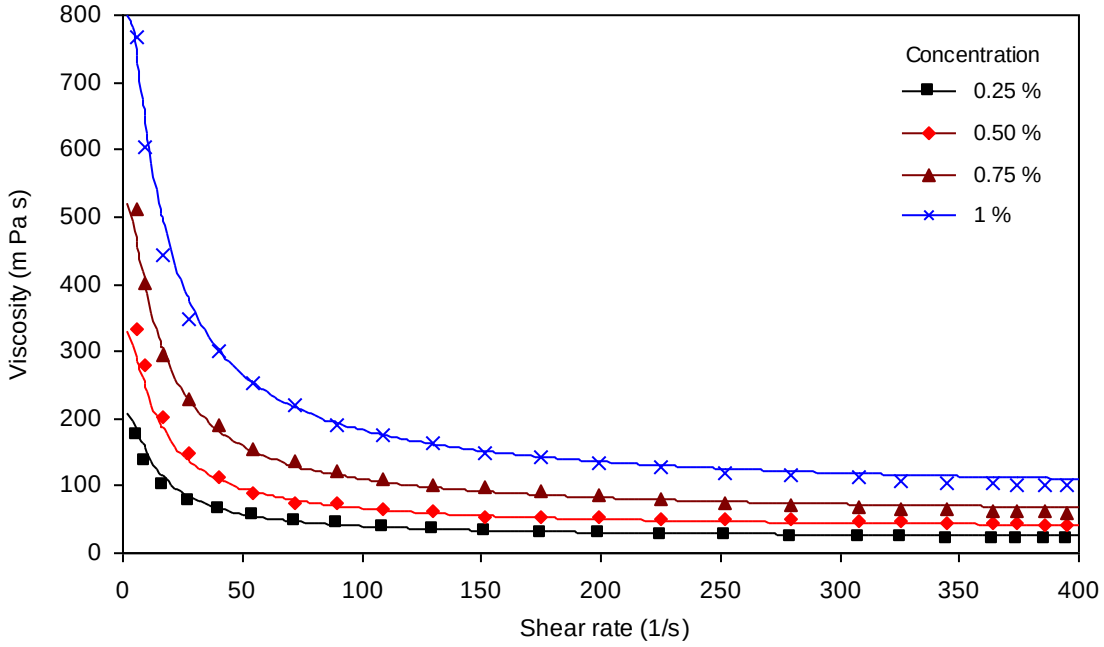

Fig 9 : Viscosity-shear rate curves where solid lines represent proposed model fit.

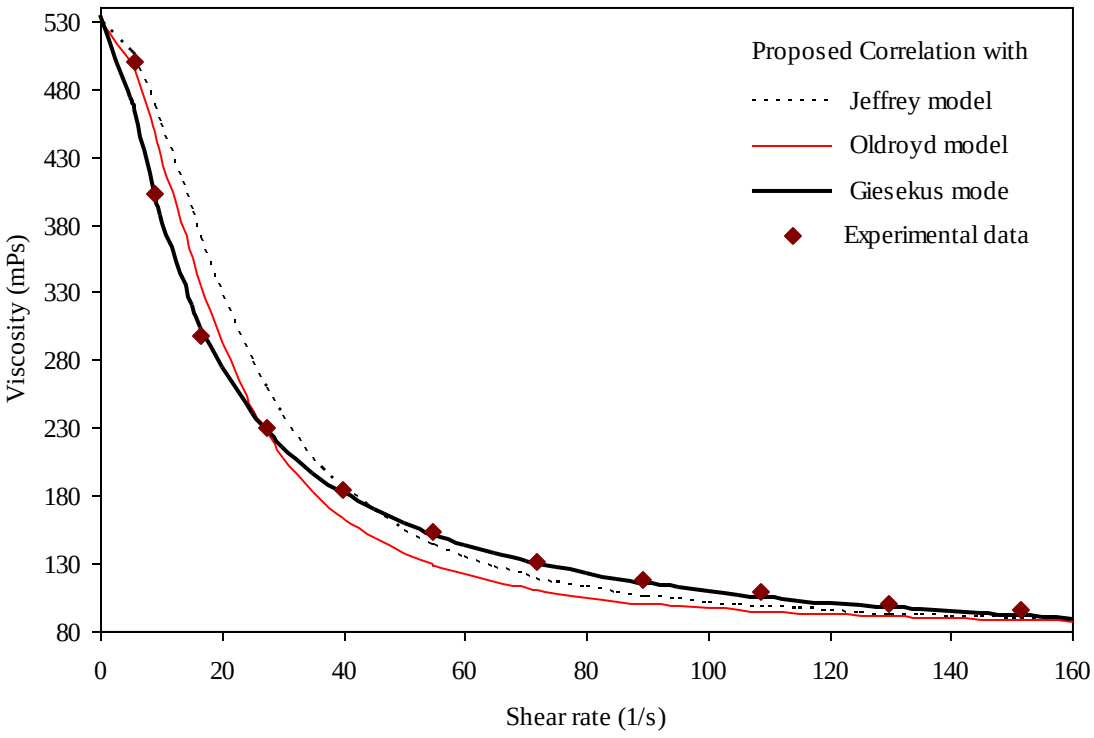

Fig 10 : Viscosity versus shear rate for PAA solution at $C = 0.75\%$, where solid and dashed lines represent the proposed correlations fit from equation 22 by using three models.

CONCLUSION

The objective of this study is to report viscosity data of PAA aqueous solution which depend on both shear rate and concentration to develop a method to correlate the viscosity data using a mathematical equation. Various viscoelastic models are reviewed for a better understanding of their applicability in determining the viscosity of PAA solution. A new shear rate and concentration dependent viscosity equation based on the Giesekus and exponential equation is introduced. This model takes into account the effects of shear thinning non-Newtonian characteristics and concentration on the viscosity of the PAA solution. The proposed equation fits adequately the experimental data of PAA solution. Therefore, from this study we can conclude that:

- Viscosity of PAA solution depends on its concentration.
- The drop in viscosity is very sharp as shear rate increases slightly.
- Depending on the PAA concentration, different flow behaviour are observed. At low concentration (0.25%), the rheological behaviour simulate a linear relationship (Newtonian fluid) which indicates that the shear rate has less effect on viscosity. While at high concentration (1%), the shear rate has a large effect on viscosity of PAA which simulate the non-Newtonian fluids.
- Jeffrey's model is relatively simple but it cannot produce the viscosity function over a large range of the shear rate.
- Oldroyd 8-constant model can describe more rheological behaviours than Jeffrey's model but without completeness for PAA solutions.
- The best agreement between experimental and calculated apparent viscosity is obtained with the Giesekus model. The success of this model is attributed to the deformation rate dependence of its viscosity and time constants.

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